Computational experience with pseudoinversion-based training of neural networks using random projection matrices

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September 12, 2014

Summary

1. Introduction
2. Training by Pseudoinversion
3. Random Projections
4. Experimental Investigation
5. Conclusions and future works

Introduction

- Our work moves forward from some new ideas concerning the use of matrix pseudoinversion to **train** a Single Hidden Layer Feedforward Networks (SLFN): the Extreme Learning Machine (Huang et al., 2006).
- The main feature of this method is that **input weights** are randomly chosen, and never modified, while **output weights** are anatically determined by pseudoinversion.
- This **non iterative** procedure makes training very fast, but some care is required because of the known numerical instability of pseudoinversion.

Introduction

- We have introduced a **regularisation task** to increas the numerical stability of the problem.
- The aim of our work is to find out if there is a possible **better inizialisation for the input weights**. We look at the **Random Projection** for their recent results in other fields *(Achliopts, 2001 and Bingham and Mannila, 2001)*.

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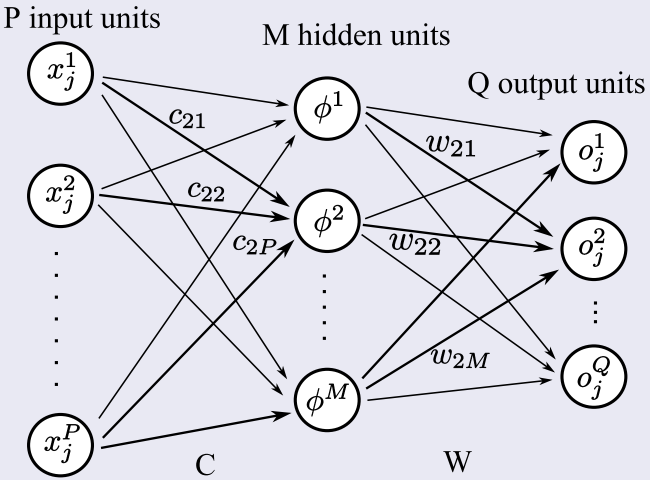
Single Layer FeedForward Network

SLFN Structure

- P input neurons
- M hidden neurons
- Q output neurons
- ϕ non-linear activation function *(sigmoid, hyperbolic tangent...)*

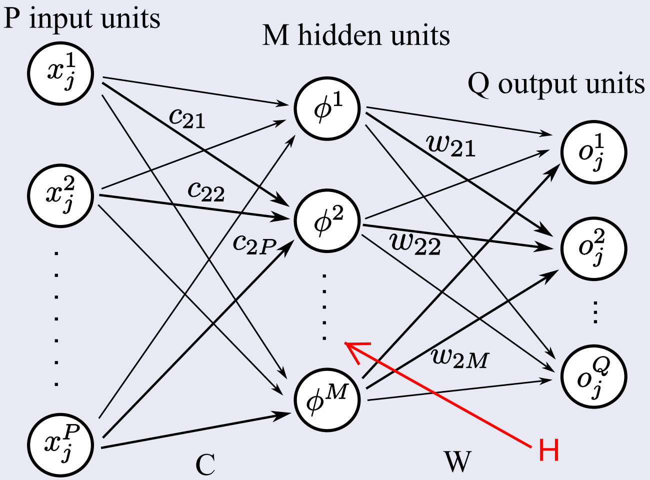
Single Layer FeedForward Network

SLFN Scheme



Single Layer FeedForward Network

SLFN Scheme



Training by Pseudoinversion

Dataset

N training samples:

$$(\mathbf{x}_i, \mathbf{t}_j) \in \mathbb{R}^P \times \mathbb{R}^Q \implies \begin{array}{l} X \in \mathbb{R}^{N \times P} (\text{input}) \\ \\ T \in \mathbb{R}^{N \times Q} (\text{desired output}) \end{array}$$

$C = (c_{ij})$ input weights

$W = (w_{ij})$ output weights

We get the linear system

$$HW = T$$

where

$$H = \phi(XC) \in \mathbb{R}^{N \times M}$$

Training by Pseudoinversion

To find the solution we consider the cost functional

$$E_D = ||HW - T||_2^2$$

The least square solution W^* is

$$W^* = H^+ T$$

H^+: Moore-Penrose pseudoinverse

Tikhonov regularisation

The problem is ill-posed $\Rightarrow$ We want to transform in a well-posed.

$$E \equiv E_D + E_R = ||HW - T||_2^2 + \lambda ||W||_2^2.$$

Training by Pseudoinversion

Compute the regularised solution $\hat{W}$

SVD decomposition of H (remeber that $H = H(C)$)

$$H = U\Sigma V^t$$

regularized solution is

$$\boxed{\hat{W} = VDU^t T,}$$

where

$$D_i = \frac{\sigma_i}{\sigma_i^2 + \lambda}.$$

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Basic Ideas on Random Projections

$$X^{RP}_{N \times K} = X_{N \times P} C_{P \times K}$$

Columns of $C_{P \times K}$ are **orthogonal versors**.

Lemma (Johnson-Lindenstrauss, 1984)

If a set of points in a vector space is randomly projected onto a selected space of suitable dimension, then the original distances between the points are approximately preserved in the new space, with only minimal distortions.

- Preserve the topological structure of the initial input space.
- Creation of a new optimal data representation in the hidden layer space, able to easy the classification/diagnosis task and to increase performance.
- Computational advantages.

Basic Ideas on Random Projections

It's difficult to obtain this matrix without a process of orthonormalisation, but these two results help us.

Theorem (Hecht-Nielsen, 2011)

In a high-dimensional space, there exists a much larger number of almost orthogonal than strictly orthogonal directions.

Proposition (Bingham and Mannila, 2001)

Vectors with random directions might be sufficiently close to orthogonality,

$$C^T C \approx I.$$

Random Projection Initialization

RP Gaussian initialization

$$c_{ij} = \mathcal{N}(0, 1)$$

RP Sparse initialization (Achiloptas, 2001)

$$c_{ij} = \sqrt{3} \cdot \begin{cases} +1 & \text{prob. } 1/6 \\ 0 & \text{prob. } 2/3 \\ -1 & \text{prob. } 1/6 \end{cases}$$

Uniform initialization (not RP)

$$c_{ij} = \mathcal{U}(-1, 1)$$

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UCI datasets characteristics

Dataset	Type	N. Instances	N. Attributes	N. Classes
Abalone	Regr.	4177	8	-
Cpu	Regr.	209	6	-
Delta Ailerons	Regr.	7129	5	-
Housing	Regr.	506	13	-
Iris	Class.	150	4	3
Diabetes	Class.	768	8	2
Wine	Class.	178	13	3
Landsat	Class.	4435	36	7

Calibration phase

Free parameters:

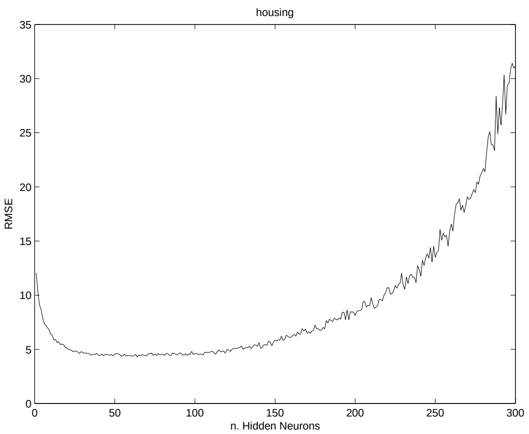
- nH: Number of Hidden neurons.
- λ: regularisation value.

Calibration

1. 100 trials of unregularised state varying number of **hidden neurons** on the validation set.
2. Fixed the size of hidden layer find the λ value of the **best performance** on the validation set.

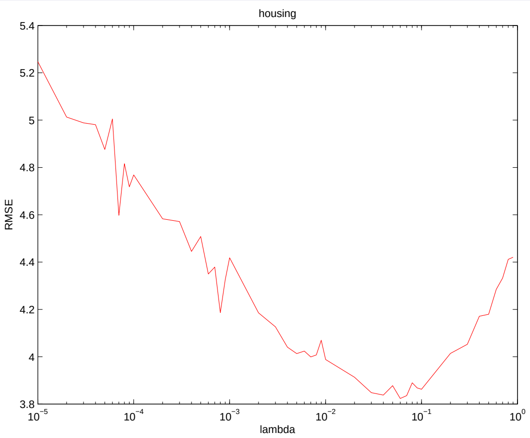
Calibration phase

Unregularized performance (validation set error)



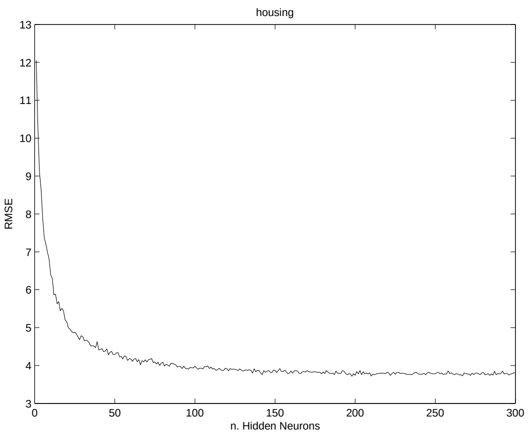
Calibration phase

Finding λ (validation set error)



Calibration phase

Regularised performance (validation set error)



Calibration phase

Calibrated parameters

	Unif.		Gauss.		Sparse	
	nH	λ	nH	λ	nH	λ
Abalone	128	3e-2	129	3e-1	118	3e-2
Mach. Cpu	98	4e-2	61	8e-1	89	5e-1
Delta Ail.	244	3e-3	272	3e-2	225	3e-3
Housing	130	8e-3	200	5e-2	180	7e-2
Iris	102	3e-4	120	3e-2	266	3e-3
Diabetes	266	3e-3	173	3e-2	192	3e-3
Wine	60	3e-2	70	2e-1	80	8e-2
Landsat	579	3e-3	600	3e-2	600	3e-3

Experimentations

Methodology

- 100 trials for each fixed size (nH, λ).
- The error is the average test set error value of 100 trials.
- In **bold** results statistically significat with confidence level of 95% for a Student's test.

Experimentations

Results

	Unif.		Gauss.		Sparse	
	Avg	StD	Avg	StD	Avg	StD
Abalone	2.165	0.004	2.169	0.009	**2.162**	0.006
Mach. Cpu	57.35	1.7	**56.85**	2.8	57.86	1.6
Delta Ail.	1.636e-4	2e-3	**1.630e-4**	4e-3	1.636e-4	2e-3
Housing	3.61	0.21	3.58	0.19	3.64	0.18
Iris	1.00	1.1	1.88	1.1	1.08	1.0
Diabetes	20.312	0.8	20.430	1.0	**20.086**	1.0
Wine	2.2542	1.5246	2.0847	1.6313	2.5593	1.5704
Landsat	10.438	0.32	**9.848**	0.30	10.394	0.32

Experimentations

Random projections based training (pseudoinversion) vs. backpropagation (tunedit.org)

Dataset	PINV		Backprop.	
	Avg.	stdDev	Avg.	stdDev
Abalone	**2.162**	0.006	2.3044	0.1908
Mach. Cpu	**56.85**	2.8	28.6673	27.3535
Delta Ail.	**1.630e-4**	4e-7	2e-3	0.0
Housing	**3.58**	0.19	4.5492	0.9517
Iris	**1.00**	1.1	1.73	0.85
Diabetes	**20.086**	1.0	26.52	2.38
Wine	**2.0847**	1.6313	3.77	0
Landsat	**9.848**	0.30	13.03	0.63

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Conclusions

- RP give **better performance** (excluse Iris) than classic uniform initialization.
- There is a **stastically significance** of the goodness of results related to RP initializations.
- RP initialization matrices is a useful tool for improving **computationally and complexity** performances.
- Direct method regularised (PINV) is **faster** then classic iterative (Backprop.).

Future works

- Obtain better performances **varying** the input weights matrix C.
- Improvement not made by a random *blind* search, but starting from an initialization $C^{(1)}$ and **moving around** it getting new matrices $C^{(k)}$.
- **Strategy**: swapping two rows, so a RP matrix with swapped rows is still a RP matrix (Local Search).

Thanks for your attention. . .